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Estimate of the decay widths of scalar mesons $(\epsilon-\epsilon')$ connection between meson-meson sigma terms and the phenomenology of scalar meson decays.

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From current algebraic techniques, PCAC approximations and a plausible conjecture, the decay widths of scalar mesons have been estimated. Using ansatz for meson-meson sigma-terms, direct connections between these terms have been derived. These relations provide the decay widths of scalar-mesons and the ratio $L\left(\frac{g_{kk\sigma_8}}{g_{\pi\pi\sigma_8}}\right)$. The results confirm the present experimental data

1. Introduction

Since Gell-Mann's original suggestions (1962, 1964) of the SU(3)⊗SU(3) structure of the strong interaction Hamiltonian, much effort has been devoted to the study of the approximate symmetries of strong interactions. The simplest assumption, due to Glashaw and Weinberg (Glashaw *et al* 1968) and Gell-Mann and others (Gell-Mann *et al* 1968), is that the symmetry breaking part of the Hamiltonian density belongs to the $(3, \bar{3})\otimes(\bar{3}, 3)$ representation of SU(3)⊗SU(3). Even then, this simple model has failed to provide crucial information about the nature of chiral symmetry breaking, to what extent chiral symmetry must be broken and what symmetry breaking mechanisms (models) should be used.

One may get answers pertaining to the questions raised above from the study of the so-called sigma-terms in meson-baryon (Hippel-Kim 1970, Cheng *et al* 1971, Reya 1974) and meson-meson (Brandt *et al* 1970, Kang 1973) scattering. The study of meson-baryon sigma-terms has attracted a great deal of attention because the present meson-baryon scattering data are, compared to meson-meson data, less controversial and ambiguous.

Although meson-meson sigma-terms have a bearing on the questions mentioned earlier, relatively fewer estimates of these-terms have been made than those of meson-baryon case. The reasons of which have previously been stated. Of the few estimates the work of Brandt and Preparata (Brandt *et al* 1970) and Kang (1973) deserves special mention. But one must admit that the above authors have made many objectionable assumptions in estimating these terms, so a further investigation of the problem is necessary.

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In the present work we aim at finding out the contributions of the decay widths of scalar meson, namely $\Gamma_{\epsilon'\pi\pi}$ and $\Gamma_{\epsilon' KK}$, to the experimental (particle Data Group 1974) sum $(\Gamma_{\epsilon'\pi\pi}+\Gamma_{\epsilon' KK}) \approx (40\pm 8)$ MeV and also investigating the values of meson-meson sigma-terms. We have not obtained unique values of these terms, instead we have put some upper and lower bounds on them. At length we discuss the phenomenological implications of the above scheme

To achieve these objectives we start with the expressions of meson-meson sigma-terms (π–π and π–K scattering) In sec. 2, following the technique of Sinha *et al* (1972) and from a plausible conjecture we obtained athe values of the decay widths of scalar-meson (ϵ and ϵ').

Using ansatz for meson-meson sigma-terms and the philosophy of Sinha *et al* (1972) a set of relations** connecting $\sigma_{\pi\pi}$ and σ_{K_π} have been obtained in sect. 3

In sub sect. 3.1 of this paper, a brief mention is made of the application of these relations which provide the decay widths that fit nicely with the recent experimental data. Our results are summarised in sect 4

2. The Decay Widths of Scalar Mesons

We write down, following the procedure of Sinha *et al* (1972), the expressions of meson-meson sigma-terms of (π–K) scattering *** only and not their values

$$\sigma_{K_\pi} = -i \int d^4Z\delta(-Z_0)e^{iqz} < \pi | [J_0^{k}(0), \delta_\mu J_\mu^{k}(Z)] | \pi >$$

$$= C'_K m_K^2 \left[\frac{1}{2\sqrt{3}} \frac{g_{\pi\pi}\sigma_8}{m\sigma_8^2} + \sqrt{\frac{2}{3}} \frac{g_{\pi\pi}\sigma_0}{m\sigma_0^2} \right] \quad \ldots \quad (1)$$

$$\sigma_{\pi K} = C'_\pi m_\pi^2 \left[\frac{1}{\sqrt{3}} \frac{g_{KK}\sigma_8}{m\sigma_8^2} + \sqrt{\frac{2}{3}} \frac{g_{KK}\sigma_0}{m\sigma_0^2} \right]. \quad \ldots \quad (2)$$

From the prescription of Glashaw & Weinberg (1968) and also that of Gell-Mann, Oakes & Renner (1968) that the symmetry breaking part of the Hamiltonian density ($H_{SB} = -u_0 - cu_0$) belongs to the $(3, \bar{3})\otimes(\bar{3}, 3)$ representation of $SU(3)\otimes SU(3)$, σ_{K_π} and $\sigma_{\pi K}$ are reduced to the following forms also,

$$\sigma_{K_\pi} = \frac{1}{3}(\sqrt{2}-c/2) [< \pi | \sqrt{2}u_0 - \tfrac{1}{2}u_8 | \pi >] \quad (3)$$

$$\sigma_{\pi K} = \frac{1}{3}(c+\sqrt{2}) [< K | \sqrt{2}u_0 + u_8 | K >] \quad \ldots \quad (4)$$

** From the expressions of σ_{KK} and $\sigma_{K\pi}$ (where Kaons are reduced) and using ansatz (see sect 3) for $\sigma_{KK} = \lambda_3 m_\pi^2$, a set of similar such relations can be obtained

*** Our notation is such that in $\sigma_{\pi K}$, pions are reduced and in $\sigma_{K\pi}$, Kaons are reduced i.e. PCAC relation is used for the meson appearing at the first place of the subscript.

From eqs. (1) to (4) it is clear that the terms $g_{\pi\pi\sigma_i}$ evolve from the terms $<P|j_i|P>$ which transform like $<P|u_i|P>$, so making use of the conjecture* $<P|j_i|P> \propto <P|u_i|P>$ where $j_i = (\square + m\sigma_i^2)\sigma_i$ we get the PPS (unmixed) coupling constants :

$$g_{\pi\pi\sigma_i} = \sqrt{\frac{2}{3}}\,\frac{(\sqrt{2}-c/2)}{(c+\sqrt{2})}\,\frac{1}{c_K}\,\frac{m_\pi^2}{m_K^2}\,W_i \qquad \ldots \quad (5)$$

$$i = 0, 8$$

$$g_{KK\sigma_i} = \frac{1}{2\sqrt{3}}\,\frac{c+\sqrt{2}}{\sqrt{2}-c/2}\,\frac{m_K^2}{c_\pi m_\pi^2}\,W_i^1 \qquad \ldots \quad (6)$$

with

$$W_0 = \frac{W_0^1}{2\sqrt{2}}\, m_{\sigma_0}^2, \qquad \sqrt{2}W_8 = W_8^1\, m_{\sigma_8}^2.$$

The effect of the $(\epsilon - \epsilon')$ mixing angle on the above coupling constants is taken by using the usual relations,

$$\left.\begin{aligned} \sigma_8 &= \epsilon \sin\theta + \epsilon' \cos\theta \\ \sigma_0 &= \epsilon \cos\theta - \epsilon' \sin\theta \end{aligned}\right\} \qquad \ldots \quad (7)$$

The expressions for $g_{pp\epsilon}$ and $g_{pp\epsilon'}$ can easily be found out from eqs. (5), (6) and (7).

In all our numerical estimations we have used the value (Sinha *et al* 1974) of $c \left(= \sqrt{2}\,\frac{c_\pi m_\pi - c_K m_K}{c_\pi m_\pi + 2 c_K m_K} \right)$ to be equal ot $-0{\cdot}98$.

This expression for c was obtained by the use of Kaon and Pion PCAC relations only. The interesting feature in the derivation of this relation is that the assumption of $c_\pi = c_K$ and $<0|u_8|0> = 0$ of GMOR scheme are not required.

At present the mass of ϵ lies between (600–700) MeV and hence the coupling constants cannot be uniquely determined. The variation of $\Gamma_{\epsilon'\pi\pi}$ and $\Gamma_{\epsilon'KK}$ is given in table 1 for different values of m_ϵ along with $(\epsilon - \epsilon')$ mixing angles.

From the above table it is clear that the variation in the sum $\Gamma_{\epsilon'\pi\pi} + \Gamma_{\epsilon'KK}$ against m_ϵ is small and the two together are very close to the experimental findings (Particle Data Group 1974) (40+8) MeV. In table 1, all the mixing angles along with the corresponding mass refer to $\Gamma_{\epsilon\pi\pi} = 600$ MeV, the present experimental lower limit.

* Eqs. (1)–(4) provide only two independent equations in four unknown $g_{\pi\pi\sigma_i}$ and $g_{KK}{}^i$ (where $i = 0, 8$). With the help of the above conjecture one can equate the zeroth and eight parts of eq (1) with the corresponding parts of eq. (3) to get $g_{\pi\pi\sigma_0}$ etc.

Table 1. $(\epsilon-\epsilon')$ mixing angles and the decay-widths while $\Gamma_{\epsilon\pi_\pi} = 600$ MeV.

m_{ϵ} in MeV	600	625	650	675	700
$(\epsilon-\epsilon')$ Mixing angle	59°24′	54°51′ 55°00′	50°30′	46°20′	42°00′
$\Gamma_{\epsilon'\pi\pi}$ (MeV)	1·59	4·30	8·61	14·72	22·81
$\Gamma_{\epsilon'kk}$ (MeV)	40·65	34·18	26·88	19·23	11·85

Further it has been found experimentally (Particle Data Group 1974) that ϵ' is associated with a sharp fall in the $(\pi-\pi)$ s-wave elasticity. The inelastic channel responsible for this drop is π-$\pi \rightarrow \bar{K}K$. As ϵ' couples strongly to $K\bar{K}$ system, the KK isoscalar s-wave scattering comes out to be quite high

In view of the above observation our analysis favours $m_\epsilon = 600$ MeV because the ratio $\Gamma_{\epsilon'\pi\pi}/\Gamma_{\epsilon'kk}$ increases with the increase in the mass of ϵ.

3 Connections Between Meson-Meson Sigma-Terms

We start with the expression of $\sigma_{\pi\pi}$ and $\sigma_{K\pi}$ (the expression of $\sigma_{k\pi}$ has been listed in the preceding sections) :

$$\sigma_{\pi\pi} = c_0 m_\pi^2 \left[\frac{1}{\sqrt{3}} \frac{g_{\pi\pi\sigma_8}}{m_{\sigma_8}^2} + \sqrt{\frac{2}{3}} \frac{g_{\pi\pi\sigma_0}}{m_{\sigma_0}^2} \right] \tag{8}$$

For numerical estimation of these terms we require the values of $g_{\pi\pi\sigma_i}$'s ($i = 0, 8$), but these coupling constants have not yet been previously determined, theoretically or experimentally. So instead of determining these sigma-terms directly from eqs. (1) and (8), we make the ansatz that the values of $\sigma_{\pi\pi}$ and $\sigma_{\pi k}$ are $\lambda_1 m_\pi^2$ and $\lambda_2 m_\pi^2$ respectively.

The reason for making such ansatz is as follows :

With the values of $\sigma_{\pi\pi}$ and $\sigma_{\pi k}$ of Brandt *et al* (1970), and Kang (1973) as input, eqs. (1) and (8) are solved for $g_{\pi\pi\sigma_i}$'s from which we get the corresponding decay widths.

The results thus obtained differ widely from the present experimental data (Particle Data Group 1974).

Following the ansatz, eqs. (1) and (8) can be written as

$$\lambda_2 = -\frac{\lambda_1}{A} + 2\frac{B}{A}\frac{g_{\pi\pi\sigma_0}}{m_{\sigma_0}^{2}} \tag{9a}$$

$$\lambda_2 = 2\frac{\lambda_1}{A} - \frac{1}{\sqrt{2}}\frac{B}{A}\frac{g_{\pi\pi\sigma_8}}{m_{\sigma_8}^{2}} \tag{9b}$$

where

$$A = 2\frac{c_{\sigma}}{c_K}\frac{m_{\sigma}^2}{m_K^2}, \qquad B = \sqrt{\frac{3}{2}}\, c.$$

With $\Gamma_{\pi\pi\sigma_0} = 600$ MeV, the experimental (Particle Data Group 1974) lower limit, as input, eqs. (9a) and (9b) reduce to

$$\lambda_2 = -\frac{\lambda_1}{A} + 9{\cdot}537 \qquad \text{(10a)}$$

$$\lambda_2 = 6{\cdot}358 - \frac{\sqrt{2}}{3}\frac{B}{A}\frac{g_{\pi\pi\sigma_8}}{m_{\sigma_8}^{2}} \qquad \ldots \text{(10b)}$$

and

$$\lambda_1 = 0{\cdot}4034 + \frac{\sqrt{2}}{3} B \frac{g_{\pi\pi\sigma_8}}{m_{\sigma_8}^{2}}. \qquad .. \text{(10c)}$$

Since λ's as shown in appendix, and $g_{\pi\pi\sigma_i}$'s are positive (see sect. 2), the maximum and minimum values of λ_2 are 6·358 and zero, the maximum and the minimum values of λ_1 are 1·211 and 0·4034 respectively.

Putting $\lambda_1 = 1$ in eq. (10a) we get $\lambda_2 = 1{\cdot}66$ which is close to the value obtained by Kang.

Further it has been shown in appendix that for $\lambda_1 = 1$, $\lambda_2 = 1{\cdot}282$, in excellent agreement with work of Kang (1973).

3.1 *Phenomenology of scalar meson decays*

It is evident from eq. (10c), that λ_1 is minimum (for $\Gamma_{\pi\pi\sigma_8} = 600$ MeV) if $g_{\pi\pi\sigma_8} = 0$ and λ_1 will increase if $g_{\pi\pi\sigma_8}$ picks up its value from zero. Further it has experimentally been revealed that $g_{\pi\pi\sigma_8}$ is small, the sum $(\Gamma_{\pi\pi\sigma_8} + \Gamma_{KK\sigma_8})$ is equal to (40 ± 8) MeV and the ratio $\alpha \left(= \frac{g_{KK\sigma_8}}{g_{\pi\pi\sigma_8}} \right)$ is of the order of 5. These results show that $\Gamma_{\pi\pi\sigma_8} = 22{\cdot}39$ MeV and $\lambda_1 = 4286 \times 10^{-4}$. If there is no experimental uncertainty for $\Gamma_{\pi\pi\sigma_0}$, λ_1 (or $\sigma_{\pi\pi}$) can accurately be determined from eq. (10c).

In view of the above discussion, the value of λ_1 is increased at stages by one percent from its minimum, and the substitution of these changed values of λ_1 in eq (10c) results in the corresponding values of $g_{\pi\pi\sigma_8}$ and the decay widths. From the knowledge of $\Gamma_{\pi\pi\sigma_8}$ and the experimental findings $(\Gamma'_{\pi\pi\sigma_8}+\Gamma_{\pi\pi\sigma_8})$ $(40+8)$ MeV, $\Gamma_{kk\sigma_8}$ and $g_{kk\sigma_8}$ can be obtained. It is to be noted that a spectrum of values of the coupling constants has been listed in table 2, but only that value will be considered which satisfy the condition $\alpha \left(= \frac{g_{kk\sigma_8}}{g_{\pi\pi\sigma_8}}\right) \approx 5$ the present experimental finding. The variation of the decay widths and the ratio α with β $(\lambda_1^{ch} - \lambda_1^{min})$ the difference between the changed and the minimum value of λ_1, is shown in table 2.

Table 2. The decay widths of scalar mesons and the ratio α, while $\Gamma_{\pi\pi\sigma 0} = 600$ MeV

β 10^6 4031	1·0	2·0	3·0	4·0	5·0	6·0
$\Gamma_{kk\sigma_8}$ (MeV)	0·5898	2·3592	5·3082	9·4368	14·745	21·2145
$\Gamma_{lk\sigma_8}$ (MeV)	43·4102	11·6408	38·6918	34·5632	29·2550	22·7885
$L\left(\frac{g_{kk\sigma_8}}{g_{\pi\pi\sigma_8}}\right)$	11·93	22·00	11·09	10·04	7·38	5·60

The above table makes it clear that for $\beta = 2224 \times 10^{-6}$, $\alpha = 5{\cdot}60$, which is close to the experimental value $L = 5{\cdot}0$, and $\sigma_{\pi\pi} = 0{\cdot}4276\ m_\pi^2$ and not m_π^2 as obtained by Brandt *et al* 1970).

The important feature of the present work is that our results, as given in table 2, are flexible enough to accommodate any value of L which the experimentalists will find in future. Once L is fixed $\sigma_{\pi\pi}$ can be determined from table 2.

4. Discussion and Conclusions

In the preceding sections we studied meson-meson sigma-terms and obtained the numerical estimations of the decay widths for scalar mesons. These results are not conclusive because they depend on the mass of the scalar mesons (m_σ) whose value is not precisely known. In the papers (Brandt *et al* 1970, Kang 1973) the value of $\sigma_{\pi\pi}$ and σ_{kk} have been estimated to be m_π^2 and $1{\cdot}1\ m_k^2$ respectively. This view has to be modified as the conclusions in sect. 3 indicate.

The set of eqs. (10b) and (10c) are very important as well as interesting, in view of the fact that if the coupling constants $g_{\pi\pi\sigma_0}$'s are accurately determined the meson-meson sigma-terms will also be precisely known and vice-versa.

We have not considered in sect 3 the effect of particle mixing for scalar mesons [σ_0 corresponds to physical particle ϵ (600 MeV)] for if $\sigma_{\pi\pi}$ and $\sigma_{\pi K}$ are accurately determined, the $g_{k\pi\sigma_0}$'s can be found out from eqs. (10a) to (10c). From the knowledge of these coupling constants, one can test whether mixing is necessary to fit the decay widths with the present experimental data.

Further it is to be mentioned that because the experimental uncertainty in the estimations of the parameters like m_ϵ, and the decay widths the results as obtained in the preceding sections, should not be taken as decisive.

Finally we hope that the present work will help to create some interest for experimental determination of the above mentioned parameters and the meson-meson sigma-terms.

Appendix

We shall outline the calculations of meson-meson sigma-terms following the philosophy of Gell-Mann, Oakes and Renner. In this model it is supposed that the SU(3)⊗SU(3) symmetry breaking hadron energy density is for the form

$$H' = u_0 + c\,u_8$$

where u_0 and u_8 being the singlet and the eight component of the octet scalar densities belonging to the $(3,\bar{3})\oplus(\bar{3},3)$ representation of su(3)⊗su(3)

In this model the axial vector current divergences are given by

$$\left.\begin{aligned} \partial_\mu A_\pi^\mu &= \left(\sqrt{\tfrac{2}{3}} + \frac{1}{\sqrt{3}}c\right) v_\pi \\ \partial_\mu A_K^\mu &= \left(\sqrt{\tfrac{2}{3}} - \frac{1}{2\sqrt{3}}c\right) v_K \end{aligned}\right\} \qquad \ldots \quad (A1)$$

The vacuum to one meson matrix element of the current divergences may be expressed in the low energy limit as

$$\langle 0|\partial_\mu A_i^\mu|P_i\rangle = c_i m_i^2 \qquad \ldots \quad (A2)$$

where c_i and m_i are the decay constant and mass of the psedosalar meson p_i

With the use of eq (A1) the expressions for $\sigma_{\pi K}$ and $\sigma_{\pi\pi}$ reduce to the following form

$$\sigma_{\pi k} = \frac{1}{2\sqrt{3}}\,\frac{1}{c_k}\,(c+\sqrt{2})\,\langle 0|v|K\rangle \qquad \ldots \quad (A3)$$

$$\sigma_{\pi\pi} = \frac{c+\sqrt{2}}{\sqrt{3}c_\pi}\,\langle 0|v_\pi|\pi\rangle. \qquad \ldots \quad (A4)$$

It is obvious from eqs. (A1), (A3) and (A4) that σ's and hence λ's are positive. Taking the ratio of eqs. (A3) and (A4) we get

$$\frac{\sigma_{\pi}}{\sigma_{k}} = \frac{2c_k}{c_\pi}\frac{<0|v_\pi|\pi>}{<0|v_K|K>} = 2\frac{c_k}{c_\pi}X \quad \dots \quad (A5)$$

where

$$X = \frac{<0|v_\pi|\pi>}{<0|v_K|K>}$$

In the soft meson limit the expressions $<0|v_\pi|\pi>$ and $<0|v_K|K>$ can be written in terms of the vacuum expectation values of u_0 and u_8, as follows

$$<0|v_\pi|\pi> = \alpha_\pi <0|v_\pi|\pi(q=0)>$$

$$= \frac{\alpha_\pi}{\sqrt{3}c_\pi}[\sqrt{2}<0|u_0|0>+<0|u_8|0>] \quad \dots \quad (A6)$$

$$<0|v_K|K> = \frac{\alpha_K}{\sqrt{3}c_K}[\sqrt{2}<0|u_0|0>-\tfrac{1}{2}<0|u_8|0>] \dots \quad (A7)$$

where $\alpha_{\pi,K}$ measure the corrections due to the soft pion and Kaon limits

Eqs. (A6) and (A7) lead to

$$X = \frac{\alpha_\pi}{\alpha_K}\frac{c_K}{c_\pi}\left(\frac{1+r}{1-r/2}\right) \quad \dots \quad (A8)$$

where $r = \frac{<0|u_8|0>}{\sqrt{2}<0|u_0|0>}$ measures the su(3) breaking of the vacuum.

It has been observed that $\alpha_\pi/\alpha_K \approx 0{\cdot}6$ and $r \approx 0{\cdot}3$ and hence eq (A8) leads to $X \sim 1/3$.

With $X \sim 1/3$ eq. (A5) reduces to

$$\frac{\lambda_1}{\lambda_2} \approx \frac{2}{3}\frac{c_k}{c_\pi}$$

Putting $\lambda_1 = 1$, λ_2 comes out to be equal to 1·282.

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